

Negation, anti-realism, and the denial defence

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Abstract Here is one argument against realism. (1) Realists are committed to the classical rules for negation. But (2) legitimate rules of inference must conserve evidence. And (3) the classical rules for negation do not conserve evidence. So (4) realism is wrong. Most realists reject 2. But it has recently been argued that if we allow denied sentences as premisses and conclusions in inferences we will be able to reject 3. And this new argument against 3 generates a new response to the anti-realist argument: keep 1 and 2, avoiding 4 by rejecting 3. My aim in this paper is to see how much work in the fight against anti-realism this new response can really do. I argue that there is a powerful objection to the response: 2 is in tension with the claim that denied sentences can be premisses and conclusions in inferences. But I show that, even given this objection, the new response has an important role to play.

Keywords Classical negation · Intuitionist attack on classical negation · Anti-realist argument from the intuitionist attack on classical negation · Denial · Realism · Anti-realism · Dummett

This paper is about the relation between an old argument against classical negation, an old attack on realism built around this argument, and a new view of the nature of inference.

The argument against classical negation is the argument from the claims that legitimate rules of inference must be rationally justifiable, and that the classical

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rules for negation do not meet this condition. I shall call this argument the ‘Negation Argument’.

The attack on realism results when the Negation Argument is combined with the claim that realists must endorse classical logic. I shall call the argument against realism generated in this way the ‘Anti-Realist Argument’.

The new view of the nature of inference is the view that denials, as well as assertions, can be premisses and conclusions in inferences, and that rules of inference should be formulated in a way which recognises this fact. For example, consider the standard classical introduction and elimination rules for ‘&’.

‘&’ Introduction

$$\frac{A \quad B}{A \& B} \quad \text{From } A \text{ and } B \text{ infer } \ulcorner A \& B \urcorner$$

‘&’ Elimination

$$\frac{A \& B}{A} \quad \frac{A \& B}{B} \quad \text{From } \ulcorner A \& B \urcorner \text{ infer } A; \text{ from } \ulcorner A \& B \urcorner \text{ infer } B.$$

These rules are ‘unilateral’ in that they deal in assertion only: they lay down which assertions are required to license an assertion of ‘ $A \& B$ ’, and which assertions an assertion of ‘ $A \& B$ ’ itself licenses. According to the new view, the right rules for ‘&’ will be ‘bilateral’. They will deal in denial as well as assertion, stating not just which assertions are required to license an assertion of ‘ $A \& B$ ’, and which assertions an assertion of ‘ $A \& B$ ’ itself licenses, but also the conditions under which we may move to a denial of ‘ $A \& B$ ’, and which moves are licensed given ‘ $A \& B$ ’ denied. So a bilateral set of rules for ‘&’ might look like this (where ‘+’ is a sign for assertion and ‘−’ a sign for denial):

Asserted ‘&’ Introduction

$$\frac{+A \quad +B}{+(A \& B)} \quad \text{From an assertion of } A \text{ and an assertion of } B, \text{ move to an assertion of } \ulcorner A \& B \urcorner$$

Asserted ‘&’ Elimination

$$\frac{+(A \& B)}{A} \quad \frac{+(A \& B)}{B} \quad \text{From an assertion of } \ulcorner A \& B \urcorner \text{ move to an assertion of } A; \text{ from an assertion of } \ulcorner A \& B \urcorner \text{ move to an assertion of } B.$$

Denied ‘&’ Introduction

$$\frac{-A}{-(A \& B)} \quad \frac{-B}{-(A \& B)} \quad \text{From a denial of } A \text{ move to a denial of } \ulcorner A \& B \urcorner; \text{ from a denial of } B \text{ move to a denial of } \ulcorner A \& B \urcorner.$$

Denied ‘&’ Elimination

$$\frac{-A \quad -B}{-(A \& B)} \quad \frac{\Phi \quad \Phi}{\Phi} \quad \text{From a denial of } \ulcorner A \& B \urcorner, \text{ and given a derivation of } \Phi \text{ from a denial of } A \text{ and a derivation of } \Phi \text{ from a denial of } B, \text{ move to } \Phi.$$

I shall call the view that the right rules of inference for reasoning in a language are bilateral ‘bilateralism’ with respect to reasoning in the language.¹

Bilateralism generates a new response to the Negation Argument. For there are bilateral rules for negation which meet the standards for justification that the proponent of the Negation Argument is supposing, and from which the ordinary

¹ Compare Rumfitt (2000, p. 797).

classical rules can be derived. And this response to the Negation Argument itself raises the possibility of a new response to the Anti-Realist Argument: a response which grants the connection between realism and classical logic; concedes the anti-realist standards for the justification of rules of inference; and saves realism anyway. This paper is about how good the prospects for this new response to the Anti-Realist Argument really are.

Here, in outline, is what I am going to suggest. Bilateralism does generate an important new response to the Anti-Realist Argument. But it is a purely defensive response: a response which can be brought to bear only once other, older, battles against the anti-realist have already been won. And the factors which restrict the power of the bilateralist response to the Anti-Realist Argument also generate restrictions, though less serious ones, on the bilateralist defence of classical negation.

I should emphasize at the outset that I am not aiming to provide a comprehensive discussion of the possible motivations for, and implications of, the claim that we should recognise denials as well as assertions as premisses and conclusions in inferences. Nor am I suggesting that the possible application of bilateralism I am going to discuss is its most important possible application, or the application which has been regarded as most important by advocates of the bilateralist approach. Rather, the spirit of my discussion is as follows. There is a powerful argument against realism from the claims that realism and classical logic stand or fall together, and that the classical rules for negation are not justifiable. Bilateralism seems to generate a new response to this argument. If this response succeeds, or even partially succeeds, it changes the landscape of the debate about anti-realism. So the question of the extent to which the response succeeds is an important question, both from the point of view of deciding how to respond to the Anti-Realist Argument, and from the point of view of developing a complete picture of the difference that the move to bilateralism might make.

The paper has three parts. Section 1 sets out the Negation Argument and the Anti-Realist Argument. Section 2 says a little about how bilateralism might be motivated, summarises the bilateralist defence of classical negation, and lays out the new response to the Anti-Realist Argument that this defence suggests. Section 3 assesses the extent to which this new response succeeds.

1 The negation argument and the anti-realist argument

According to the classical view of negation, right reasoning using the negation operator is reasoning according to the following introduction and elimination rules²:

² I follow other participants in this debate in characterising the question about the legitimacy of classical logic as a question about the legitimacy of a classical natural deduction system. See for example Dummett (1991, ch. 11); Rumfitt (2000). There is a debate about whether CNI is better formalised as the stipulation that if both B and $\neg B$ can be derived from the assumption that A , together with X , we may discharge the assumption and move to $\neg A$ from X . The alternative formulation has the advantage of not using ' \perp ' (which is itself going to require rules for use). It has the disadvantage of using ' \sim ' in the specification of the conditions under which ' \sim ' may be introduced. It is not possible to enter into the debate about rival formulations of CNI here. For a brief remark on the connection between this debate and the Negation Argument see note 5.

Classical '¬' Introduction (CNI)

[A], X	If absurdity can be derived from the assumption that A, together with additional premisses X, discharge the assumption and infer '¬ A' from X alone.
⋮	
⊥	
¬ A	

Classical '¬' Elimination (CNE)

$\frac{\sim\sim A}{A}$	From '¬¬ A' infer A
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The first part of this section sets out the argument against the classical view of negation that I want to consider (the Negation Argument). The second shows how the Negation Argument is expanded into the Anti-Realist Argument by the addition of some additional, initially plausible, premisses.

1.1 The negation argument

The Negation Argument goes like this.

1 A rule of inference is legitimate only if to take a step in an inference according to the rule is to take a step which is rationally justified.

2 The rules for a logical constant meet the condition in 1 only if they are in harmony, where 'harmony' is defined as follows ('...C...' abbreviates 'a statement whose operator of widest scope is C'):

Definition—The introduction and elimination rules for logical constant C are in 'harmony' iff any consequence derived from '...C...' using C's elimination rule would already appear in a direct derivation of the premiss from which '...C...' is derived by C's introduction rule, or can be derived directly from this premiss without using the rules for C.

and

Definition—A 'direct' derivation of a statement is a derivation of the statement which starts from atomic sentences and uses only introduction rules.³

The harmony constraint is a consequence of a very intuitive view of the role of deductive inference: the view of deductive inference as enabling us to extend what we know by manipulating the evidence we already have. To uphold this view you need to say that each sentence is associated with some determinate package of evidence: this will be the evidence that the sentence brings to a proof when it is assumed as a premiss, and the evidence that must be extractable from the evidence brought in by a proof's premisses if it is to be legitimate to move to the sentence as a conclusion. The role of the notion of 'direct verification' in the statement of the harmony constraint is to provide standard packages of evidence of this kind. Intuitively, the suggestion is that a sentence's direct verification consists in a

³ For the definitions and the claim that harmony is a requirement on legitimate rules of inference see Dummett (1991, pp. 219, 246–248; 1978, pp. 221–222).

specific arrangement of specific bits of evidence, so that the sentence may be regarded as ‘containing’ all the bits of evidence which are put together to make its direct verification. The evidence the sentence contains is then the evidence it brings to a proof when assumed as a premiss, and the evidence which must be contained in the proof’s premisses if the sentence is to be reached as a conclusion. Taking a step in a proof may be regarded as manipulating the evidence contained in the lines of the proof so far to produce a direct verification for the sentence you add to the proof when you take the step.⁴ The harmony constraint is just a way to state the claim that applications of introduction and elimination rules are to be evidence-manipulating, and so evidence-conserving, steps of this kind. An introduction rule for C conserves evidence iff the direct verification for ‘... C ...’ can be constructed out of the evidence contained in the premiss or premisses to which the rule is applied to reach ‘... C ...’. And an elimination rule for C conserves evidence iff a direct verification for ‘... C ...’ contains all the evidence you would need to construct direct verifications for the conclusion or conclusions the elimination rule takes you to. So the use of C in proofs conserves evidence iff the direct verification for a sentence you get by applying the elimination rule to ‘... C ...’ is constructible out of the evidence you already had before you applied the introduction rule to reach ‘... C ...’: the use of C in proofs conserves evidence iff C ’s introduction and elimination rules are in harmony.

But

3 The introduction and elimination rules for classical negation are not in harmony.

Here is why not in informal terms. CNI lays down that ‘ $\sim A$ ’ may be asserted iff the assertion of A leads to inconsistency. Given this introduction rule, an assertion of ‘ $\sim A$ ’ may be regarded as containing evidence for the claim that it is inconsistent to assert A . And an assertion of ‘ $\sim\sim A$ ’ (the premiss of CNE) may be regarded as containing evidence that any attempt to derive an inconsistency from A will itself be inconsistent. So an assertion of ‘ $\sim\sim A$ ’ contains evidence that it is consistent to assert A (because it contains evidence that it is inconsistent to say that it is inconsistent to assert A). But evidence for A ’s consistency falls short of evidence for A . So CNE (From ‘ $\sim\sim A$ ’ infer A) tells us to move from ‘ $\sim\sim A$ ’ to a conclusion evidence for which ‘ $\sim\sim A$ ’ does not contain.

So

4 The classical laws for negation are not legitimate.⁵

⁴ For representative discussion of the ‘inference preserves evidence’ model in Dummett see (1978, especially 307–308, 312–313; 1991, pp. 217–220). There is a hard question about whether the inference-preserves-evidence framework can be sustained in a version which does not require an appeal to the notion of ‘direct’ or ‘canonical’ verification. But I cannot discuss this question here.

⁵ Dummett (1991, p. 291). Compare Campbell (2002, pp. 196–197), Rumfitt (2000, pp. 790–791). Any combination of rules for ‘ \sim ’ which uses some form of *reductio* introduction rule, and validates proof by *reductio* (proof of A from the reduction to absurdity of ‘ $\sim A$ ’) will entail that use of ‘ \sim ’ does not conserve evidence, and so be subject to a variant of the Negation Argument. The standard intuitionist introduction rule for ‘ \sim ’ is CNI. The intuitionist elimination rule is ‘From A and ‘ $\sim A$ ’ infer \perp ’. This rule, unlike CNE,

1.2 The anti-realist argument

The Negation Argument is expanded into an argument against realism by the addition of the following two claims:

The Bivalence Characterisation of Realism—‘Realism’ with respect to a class of statements is the view that every statement in the class is either determinately true or determinately false, aside from our capacity to verify its truth or demonstrate its falsehood.⁶

The Manifestation Principle—Claims about the kinds of truth conditions that sentences in a language have must be substantiated by appeal to facts about speakers’ behaviour: to show that sentences in our language have truth conditions of a given kind we must show that there are aspects of our linguistic behaviour which manifest (depend for their justification on) our commitment to these kinds of truth conditions.⁷

The expansion proceeds as follows.

Assume the Manifestation Principle. Then there are two tasks which must be completed to show that a realist model of truth is appropriate for our language. The first is to find aspects of our linguistic behaviour which manifest our commitment to realist truth conditions. The second is to establish that these aspects of our linguistic behaviour are in good order. If there are no such aspects of our linguistic behaviour, the behavioural facts do not indicate an underlying commitment to realist truth conditions. If the aspects of our linguistic behaviour which manifest commitment to realist truth conditions are not in good order, we are committed to realist truth conditions, but our commitment is mistaken: we should eliminate uses which are not in good order; when we do, our mistaken commitment to realist truth conditions will fall away.

Now assume the Bivalence Characterisation of Realism. Given this assumption, realism with respect to a class of statements is the view that each member of the class has truth conditions which are either met or not met independently of our capacity to tell whether they are. So let us ask which aspects of our linguistic behaviour might manifest commitment to truth conditions of this kind.

Footnote 5 continued

is in harmony with CNI. So if CNI is assumed, the Negation Argument is also an argument for preferring intuitionist logic to classical logic.

⁶ See, for example Dummett (1991, p. 326; 1993, p. 56). Dummett writes that he considered retrenching from this position on the ground that there may be a distinction between ‘deep’ and ‘shallow’ rejections of bivalence, where only deep rejections are anti-realist, but ended up retaining the Bivalence Characterisation on the ground that a step away from bivalence for a class of statements is a step away from the strongest possible realist view for statements in the class, and, therefore, a form of anti-realism. See (1991, ch. 15), and compare (1993, pp. 265, 467–468). Note that for the case of vague statements Dummett proposes an amendment. See, for example, (1993, p. 468): ‘A realist must...hold that, for every vague statement, there is a range of statements giving more precise information of which a determinate one is true and the rest false’, so that failures in bivalence for vague statements are treated as arising from lack of precision in the description of a determinate reality.

⁷ For Dummett stating and defending this principle see, for example, (1991, ch. 4, especially 88–92 and 103–105).

The relevant aspects of behaviour cannot be just tendencies to assert or deny sentences. For to be right in asserting a sentence you need evidence for its truth. And to be right in denying a sentence you need evidence for its falsehood. So an account of the circumstances under which we will assert or deny sentences cannot reveal whether we are committed to the claim that a sentence has a truth value even if there is no evidence in principle available to us which would establish what this truth value is. If there are aspects of our linguistic behaviour which indicate commitment to realist truth conditions these must be aspects of our inferential behaviour.

And the most plausible candidate for inferential behaviour which displays commitment to realist truth conditions is reasoning according to classical negation. The canonical proof of $\lceil \sim A \rceil$ using classical negation will start by supposing A , derive an absurdity, and move to $\lceil \sim A \rceil$ by CNI. And the canonical proof of A using classical negation will start by supposing $\lceil \sim A \rceil$, derive an absurdity, move to $\lceil \sim \sim A \rceil$ (by CNI), then move to A by CNE. If $\lceil \sim A \rceil$ is taken to be true iff A is false, these two patterns of reasoning combine to display commitment to the view that every statement is either true or false. For the absurdity of A 's falsehood entails A 's truth and the absurdity of A 's truth entails A 's falsehood iff it is guaranteed that A is either true or false.⁸

So the Manifestation Principle and the Bivalence Characterisation of Realism generate a connection between realism and classical negation. Given the Bivalence Characterisation, it is by reasoning according to the classical rules for negation that we manifest our commitment to realist truth conditions. So given the Manifestation Principle, realism with respect to a subject matter entails commitment to the legitimacy of reasoning about the subject matter according to these rules.⁹

Combining the pieces now on the table, the Anti-Realist Argument can be summarised as follows. The main pattern of linguistic use which manifests our commitment to realist truth conditions—reasoning according to the classical rules for negation—is illegitimate. So realism must be abandoned.

1.3 Standard responses to the negation argument and the anti-realist argument

Philosophers have explored a range of responses to both the Negation Argument and the Anti-Realist Argument. I shall give a brief survey of the main responses that have been made in each case.

⁸ Note that just upholding classical logic is not enough to manifest commitment to bivalence. For example, consider supervaluationism about vague predicates (the view that a sentence containing a vague predicate is (absolutely) true iff true under every precisification and (absolutely) false iff false under every precisification). A supervaluationist can uphold all classical theorems (because a classical theorem will come out as true under every precisification) while denying bivalence (because many sentences containing vague predicates will come out true under some precisifications but false under others). Deployment of classical laws does manifest commitment to the claim that every sentence is either true or false if the notion of absolute truth commutes with both negation and disjunction (so that 'It is not true that A ' is equivalent to 'It is true that not A ', and 'It is true that $(A$ or $B)$ ' is equivalent to 'It is true that A or it is true that B '). Compare Dummett (1991, pp. 74–75). These conditions combine with $\lceil A \vee \sim A \rceil$ to entail that there are only two truth values, and $\lceil \sim A \rceil$ has one iff A has the other, which is abbreviated in the text to ' $\lceil \sim A \rceil$ is taken to be true iff A is false'. So Dummett's claim is that reasoning according to CNI and CNE manifests commitment to bivalence iff embedded in this wider inferential environment.

⁹ Compare Dummett (1993, p. 64; 1978, pp. 316, 216–218).

Most philosophers wanting to overturn the Negation Argument reject either 1 (the claim that legitimate logical laws require rational justification) or 2 (the claim that the rules governing an operator are rationally justified only if they are harmonious). For example, some philosophers argue that 1 should be rejected because the act of taking a step in an inference is too fundamental to be rationally justifiable.¹⁰ The traditional objection to 2 is that harmony is not a necessary condition for the rationality of rules of inference because a constant's rules of inference are justified by appeal to its truth table. A philosopher mounting this objection will point to the classical truth table for ' \sim ',

A	$\sim A$
T	F
F	T

and argue as follows. 'CNI is justified because if A is absurd it cannot be true, and the truth table guarantees that if A is not true, ' $\sim A$ ' is true. CNE is justified because the truth table guarantees that if ' $\sim\sim A$ ' is true then A is true too.'¹¹

The standard realist responses to the Anti-Realist Argument fall into two groups.

Firstly, there are responses which seek to overturn the Negation Argument, adopting one of the strategies just described.¹²

Secondly, there are responses which attack the argument for the connection between realism and classical negation. For example, some philosophers have attacked this argument by attacking the Bivalence Characterisation of Realism. According to one line of response in this group,¹³ the Bivalence Characterisation results from a mistake in tightening up the intuitive characterisation of realism. According to the intuitive characterisation, realism about some aspect of the world is the view that how things are with respect to this aspect of the world does not depend on our capacity to tell how they are. One step towards tightening up this intuitive characterisation is to say that our statements about this aspect of the world have verification-transcendent truth conditions—truth conditions which may obtain or not, independently of our capacity to tell whether they do. This step takes us to the characterisation of realism about an aspect of the world in terms of commitment

¹⁰ The suggestion that basic logical beliefs are 'default reasonable' is a view of this kind. See Boghossian (2000, pp. 238–240) for a summary.

¹¹ See for example Campbell (2002, pp. 101–104, 198; 2007, p. 12). Rumfitt (2000, pp. 785–787) argues against this kind of position on the ground that a justification for a rule of inference must be concerned not only with the combinations of premisses and conclusion that it treats as legitimate but with the path from premisses to conclusion that it lays down.

¹² See for example Campbell (2002, pp. 101–104; 2007, Sect. 5).

¹³ The following line of thought forms part of McDowell's attack on the Bivalence Characterisation in (1998)—see especially Sect. 6 of that paper. Other philosophers have given other arguments against the Bivalence Characterisation. For a summary of such arguments see Hale (1997, pp. 274, 287–288). For extended discussion of the relations between realism, classical logic, and the Principle of Bivalence see Wright (1993, pp. 433–457). See note 6 for references to some places where Dummett defends the Bivalence Characterisation from these attacks.

to allowing that a statement about this aspect of the world might be true or false independently of our capacity to verify or falsify it. Advocates of this response to the Anti-Realist Argument say that the Bivalence Characterisation results from failure to recognise that the intuitive characterisation of realism should be precisified into this weak claim, rather than the much stronger claim that the Bivalence Characterisation embodies. (Note that if you take this line with respect to the Anti-Realist Argument and accept the Manifestation Principle you still need to find linguistic behaviours which manifest our commitment to the possibility that a statement might be true though we cannot tell whether it is.)

An alternative strategy is to deny the Manifestation Principle. For example, this is the tactic recommended by ‘deflationism’ about truth—the view that our grasp of the notion of truth is exhausted by our preparedness to accept any instance of the schema ‘ p is true iff p ’. From the deflationist point of view, the debate about whether our linguistic behaviour ‘manifests’ grasp of truth conditions of one kind or another is just misguided.¹⁴

Yet another possibility is to accept the Manifestation Principle, but argue that there are aspects of our linguistic behaviour other than inference according to classical logic which manifest our grasp of realist truth conditions.¹⁵

It is not possible to assess these standard responses to the Negation Argument and the Anti-Realist Argument here. But at first sight, bilateralism seems to promise responses to the Negation Argument and the Anti-Realist Argument which have a better chance of silencing the anti-realist than any of these standard moves. For bilateralism seems to offer a way to overturn the Negation Argument while granting the inference-preserves-evidence framework that generates it. And, given this response to the Negation Argument, a bilateralist seems to be in a position to grant the connection between realism and classical negation, and save realism anyway. So bilateralism seems to put the realist in a position to accept whatever may be appealing about the assumptions which generate the Negation Argument and the Anti-Realist Argument, and remain neutral about whatever is unappealing in these assumptions, pointing out to the logical revisionist and the anti-realist that even if all of their assumptions were granted their conclusions still would not follow.

The next section shows how the bilateralist responses to the Negation Argument and the Anti-Realist Argument work.

¹⁴ Compare Horwich (1990, p. 8): ‘What I am claiming on behalf of the minimalist conception of truth is not that it, by itself, will engender realism or anti-realism; but rather that it will make it easier for us to see that the central aspects of the realism debate have nothing to do with truth’. Note that deflationists will also reject the inference-preserves-evidence framework that generates the Negation Argument. See Field (1994, Sect. 3) for deflationist derivations of the introduction and elimination rules for classical logical constants.

¹⁵ For example, Dorothy Edgington argues that our grasp of realist truth conditions is manifested by our deployment of conditionals with untestable antecedents and testable consequents. See Edgington (1985, pp. 44–46).

2 Bilateralism, the negation argument, and the anti-realist argument

‘Bilateralism’ about reasoning in a language is the view that right rules of reasoning in the language are rules which govern not just how asserted sentences might be deployed in inferences, but also the inferential use of these same sentences denied. The first part of this section sets out one central motivation for bilateralism. The second states the new defence against the Negation Argument that bilateralism generates, and sets out the new response to the Anti-Realist Argument that this defence against the Negation Argument seems to promise.

2.1 Motivating bilateralism

One central motivation for bilateralism¹⁶ is the apparent independence, for many sentences, of grounds for denial from grounds for assertion. For example, it is plausible that sentences about the past are sentences of this kind. You could establish that all the evidence about the appearance of some historical figure has been destroyed. In this case you will have established that it will never be right to assert that this person was snub-nosed. But it seems wrong to say that you will also have established that it is right to deny that the person was snub-nosed. So for sentences about the past, it seems that grounds for denial are ‘independent’ from those for assertion in that the grounds required for denying a sentence may exceed those for not asserting it.¹⁷

Let *L* be a language fragment for which the grounds for denial are ‘independent’ from those for assertion in this sense. So if *A* is a sentence in *L*, establishing that there are no grounds to assert *A* does not establish that there are grounds to deny *A*. Given that denial is independent from assertion in *L*, it seems wrong to say that inferences in *L* will involve only asserted sentences. For consider the following inference (where ‘*A*? Yes!’ expresses acceptance of *A* as true, and so is a form of assertion of *A*, and ‘*A*? No!’ expresses rejection of *A* as not true, and so is a form of denial of *A*)¹⁸:

¹⁶ See Smiley (1996, p. 1) and Rumfitt (2000, pp. 178–179). What follows gives only a basic account of one motivation for bilateralism. There are many others. For example, Smiley (1996) and Rumfitt (1997) explore the move to bilateralism as a way to secure the result that the meanings of the logical constants used in a calculus are given by the introduction and elimination rules they have in a sound and complete deduction system for the calculus. Restall (2005) presents a range of motivations for bilateralism. Price (1983, p. 169) gives another motivation.

¹⁷ It is important to distinguish this claim about the independence of denial from assertion from the claim that denial is ‘independent from’ assertion in that *A*’s denial is not equivalent to the assertion of its negation. For this alternative independence claim see Parsons (1984, p. 139), Tappenden (1999, Sect. 1) and Restall (2005, p. 2).

¹⁸ This example originates with Frege’s discussion of ‘the Frege Point’—the claim that denial must be collapsed into the assertion of negated sentences in order to account for the validity of the inference from the denial of ‘*p*’ and the assertion of ‘If not *p* then not *q*’ to the assertion of ‘Not *q*’. See (1984, pp. 384–385). The bilateralist response to this point is that the validity of inferences involving denials is to be accounted for by (bilateralist) rules governing how assertions and denials interact, and that Frege’s claim that it is more ‘economical’ to deal in assertion alone is misguided. See Smiley (1996, pp. 2–4).

If the accused was in Berlin at the time of the murder could he be guilty? No!
Was the accused in Berlin at the time of the murder? Yes!

Could the accused be guilty? No!

On the face of it, this is an inference involving both assertions and denials. It is possible to transform it into one involving assertions only:

If the accused was in Berlin at the time of the murder is his guilt impossible? Yes!

Was the accused in Berlin at the time of the murder? Yes!

Is the accused's guilt impossible? Yes!

But the claim that the initial argument is improved by this or some other transformation into an assertions-only form rests on the assumption that denials are second-rate speech acts, parasitic on assertions, so to be eliminated from inferences where possible. And if the correctness conditions for denials are not parasitic on those for assertions, this assumption is wrong. Given the independence of denial from assertion in L, it is plausible that a right account of reasoning in L should recognise that when we reason using sentences in L we do so using both assertions and denials. So a right account of which moves using sentences in L are legitimate should be a bilateral account: it should provide rules for reasoning not just with asserted sentences, but also with these same sentences denied.

Philosophers interested in developing bilateral models of reasoning have said that a bilateral model is appropriate for any class of statements within which the conditions for the correctness of denial are distinct from those for the incorrectness of assertion.¹⁹

2.2 Bilateralism, the negation argument, and the anti-realist negation argument

Bilateralism yields the following response to the argument against classical negation set out in Section 1.1.²⁰

Consider this quotation from Frege:

To declare false the thought that Peter did not come to Rome is to assert that Peter came to Rome. We could declare it false by inserting a second 'not' and saying 'Peter did not not come to Rome' or 'It is not true that Peter did not come to Rome'.... two negatives cancel one another out. If we take the opposite of the opposite of something, we have what we began with. [1979, p. 149]

In this passage Frege is expressing the classical conception of the negation operator as a 'logical switch' which toggles back and forth between a sentence and its opposite, so that prefixing ' \sim ' to A takes you to a sentence which is correctly asserted iff A is correctly denied (declared false), correctly denied iff A is correctly

¹⁹ See for example Smiley (1996), Rumfitt (2000, p. 819) and Restall (2005, p. 2).

²⁰ This response to the Negation Argument is modelled on elements of Rumfitt (2000, Sects. 5–7). Price and Restall offer alternative bilateralist defences of classical negation. I focus on Rumfitt's defence because I take it to be dialectically the strongest against the Negation Argument as I have stated it. It is not possible to argue for this comparative point here.

asserted, and from which you can get back to A by applying the operator again. In a bilateral system, the rules which capture this conception look like this²¹:

$\frac{+\sim \text{Introduction}}{-A}$	$\frac{+\sim \text{Elimination}}{+(\sim A)}$	$\frac{-\sim \text{Introduction}}{+A}$	$\frac{-\sim \text{Elimination}}{-(\sim A)}$
$\frac{-A}{+(\sim A)}$	$\frac{+(\sim A)}{-A}$	$\frac{+A}{-(\sim A)}$	$\frac{-(\sim A)}{+A}$

Given these four rules, prefixing ‘ \sim ’ to A takes you to a sentence which is A’s ‘opposite’ in what looks like Frege’s sense: a sentence which may be asserted iff A may be denied (‘declared false’), and denied iff A may be asserted.

But these rules are rationally justifiable relative to the standards that the Negation Argument assumes. And they are rules from which the traditional unilateral rules for classical negation can be derived. So in a bilateral system inference using the traditional rules for classical negation meets the standards for the legitimacy of rules of inference on which the Negation Argument is built.

The claim that the proposed bilateral rules for ‘ \sim ’ are rationally justifiable relative to the standards the Negation Argument assumes is trivial. For it is trivial that the proposed pairs of introduction and elimination rules are in harmony. In each case, the conclusion of the elimination rule is the premiss of the corresponding introduction rule. So the elimination rule enables you to move from an asserted or denied sentence back to the premiss from which it is derived: in applying the elimination rule you are, trivially, just unpacking what the introduction rule guarantees is there to be unpacked.²²

The claim that the proposed rules for ‘ \sim ’ validate inferences according to the traditional rules for classical negation follows like this. First, consider CNE. The bilateral rules yield

- (1) $\vdash +\sim\sim A$ [assumption]
- (2) $\vdash -\sim A$ [from 1 by $+\sim E$]
- (3) $\vdash +A$ [from 2 by $-\sim E$].

²¹ Rumfitt (2000, p. 802; 2002, p. 307). See (2000, pp. 800–802) for bilateral rules for the other sentential operators. Note that these rules also have the advantage of neither using ‘ \perp ’, nor using ‘ \sim ’ to specify the conditions for its own introduction.

²² Rumfitt demonstrates that the bilateral rules also satisfy a requirement for rules to be in good order which is specific to bilateral reasoning systems. This is the demand for coordination, where (i) assertion and denial are ‘coordinated’ for a sentence iff assertion and denial of the sentence are never correct simultaneously; and (ii) the rules for a logical operator are ‘coordinated’ iff they ensure that assertion and denial are coordinated for a complex sentence formed using the operator if they are coordinated for the simpler sentences that the complex sentence contains. Gibbard (2002, pp. 299–300) objects that there are coordinated bilateral sets of rules for ‘ \sim ’ other than Rumfitt’s rules. I take Rumfitt to be right in replying (2002, p. 309) that even if this were the case it would not count against his claim that if bilateralism is assumed a proof-theoretic justification for the classical ‘ \sim ’ rules is available: showing that a set of rules meets the requirements for good order does not require showing that these are the only rules which meet the requirements. Rumfitt also argues that Gibbard’s proposed alternative rules could not be taken to fix the sense of ‘ \sim ’ as used in L because they do not fix conditions for denying a negated sentence (2002, pp. 310–311).

So the bilateral rules allow $\ulcorner +A \urcorner$ to be derived from $\ulcorner +\sim\sim A \urcorner$: CNE is a derived rule of the bilateral system.²³ To derive CNI we need the following structural principle²⁴:

$$+RAA \quad \text{If } X, +A \vdash +B \text{ and } X, +A \vdash -B, X \vdash -A$$

+RAA entails that if absurdity can be derived from X and $\ulcorner +A \urcorner$, X entails $\ulcorner -A \urcorner$. And an application of $\ulcorner +\sim I \urcorner$ to $\ulcorner -A \urcorner$ will yield $\ulcorner +\sim A \urcorner$. So if absurdity can be derived from X and $\ulcorner +A \urcorner$, we can move from X to $\ulcorner +\sim A \urcorner$. Given +RAA, CNI comes out as a derived rule of the bilateral system.

The bilateralist response to the Negation Argument can be summarised like this. If it is assumed that rules of inference must be rules laying down which transitions from assertions to assertions are licensed, the classical view of negation is inconsistent with the inference-preserves-evidence framework. For the only possible introduction rules for $\ulcorner \sim \urcorner$ which conform to this assumption are *reductio* rules: rules laying down that $\ulcorner \sim A \urcorner$ may be asserted iff the assertion of A leads to an inconsistency. And this kind of introduction rule cannot be in harmony with CNE. Given a *reductio* introduction rule, the evidence contained by an assertion of $\ulcorner \sim\sim A \urcorner$ is evidence for the inconsistency of the claim that the assertion of A leads to inconsistency. So, given a *reductio* introduction rule, the most that can be unpacked from $\ulcorner \sim\sim A \urcorner$ is the conclusion that asserting A does not lead to inconsistency. But CNE tells us to move from $\ulcorner \sim\sim A \urcorner$ to A . This is to unpack from $\ulcorner \sim\sim A \urcorner$ a conclusion that a *reductio* introduction rule cannot guarantee is there for the unpacking. However, this argument against the classical view of negation is only as strong as its initial assumption: the assumption that rules of inference deal in assertion alone. If we allow rules of inference which deal in denial as well as assertion, we can define the conditions for asserting $\ulcorner \sim A \urcorner$ directly in terms of those for A 's denial. And with the tie between negation and inconsistency gone, the argument for the claim that the move from $\ulcorner \sim\sim A \urcorner$ to A constitutes unpacking something not there to be unpacked falls away.

With the bilateralist response to the Negation Argument in place, it is trivial to formulate a bilateralist response to the Anti-Realist Argument.²⁵ Recall that the Anti-Realist Argument goes like this:

- 1 Realism about the subject matter of L is the view that every sentence in L is either true or false independently of our capacity to verify or falsify it. [The Bivalence Characterisation of Realism.]
- 2 A claim that sentences in L have truth conditions of a given kind (for example, realist truth conditions) is warranted iff speakers use sentences of L in ways which are legitimate, and which manifest commitment to these kinds of truth conditions. [The Manifestation Principle.]

²³ Rumfitt (2000, p. 803).

²⁴ Compare Smiley (1996, p. 5). \neg RAA would be 'If $X, -A \vdash +B$ and $X, -A \vdash -B, X \vdash +A$ '. Smiley combines the two principles into one using a 'polarity reversing' operator. Rumfitt endorses the combined 'Smilean *Reductio*' at (2000, p. 804).

²⁵ I should stress that I am not suggesting that any of the proponents of bilateralism I have referred to in this paper actually does extend the response to the Negation Argument into a response to the Anti-Realist Argument in this way.

-
- 3 Commitment to realist truth conditions is manifested by reasoning according to the classical rules for negation. [From 1]
but
 - 4 Legitimate rules of inference must preserve evidence.
and
 - 5 The classical rules for negation do not preserve evidence.
so
 - 6 Realism must be rejected. [From 2, 3, 4, 5]

Section 1.3 summarised the standard responses to this argument: reject 1; reject 2; reject 3; reject 4. The bilateralist response to the Negation Argument raises the hope of a radical new tactic for the realist: silence the anti-realist once and for all by showing that even if 1–4 are all granted the argument fails because 5 is false

If 1–3 are all granted, and 4 is strengthened into a necessary and sufficient condition for the legitimacy of rules of inference, the bilateralist response to the Negation Argument also seems to offer a new test for realism with respect to any given subject matter. If a bilateral model of reasoning is right for L, reasoning according to CNI and CNE in L meets the inference-preservation standard. If this standard is sufficient as well as necessary for legitimacy, it follows that if a bilateral model of reasoning is right for L, reasoning according to CNI and CNE is legitimate in L. In this case, given 1, 2, and 3, we can be realists about L's subject matter. And if this claim is combined with the bilateralists' suggestion about when bilateral models are appropriate, we get the following test for realism: realism about L's subject matter is legitimate iff the conditions for denying sentences in L outrun those for not asserting them.

So bilateralism seems to hold the promise of a new response to the Anti-Realist Argument, and a new perspective on how to decide whether we should be realists about any given subject matter. The next section is about the extent to which it is going to be possible to deliver on this initial promise.

3 Assessing the bilateralist responses

3.1 Three obstacles to bilateralism

On the face of it there are three immediate obstacles that a proponent of bilateralism must overcome.

The first is the unclarity of the notion of the 'independence' of conditions for correctness of denial from those for the incorrectness of assertion.

The second is the problem of what to say about mixed cases. Suppose that p comes from a class of statements for which a bilateral model of reasoning is appropriate. And suppose that q comes from a class which requires a unilateral model. Then what should we say about 'p & q'?

The third problem concerns whether the speech act of denial can do the job that bilateralism demands of it. Consider the following examples

- (1) Did Homer write the *Republic*? No! Homer (a guy who actually existed and wrote the *Odyssey* and the *Iliad*) did not also write the *Republic*.

- (2) Did Homer write the *Iliad*? No! Actually Homer did not exist.
- (3) Is Homer a valid form of syllogism? No! If Homer existed he was a person, not a form of syllogism.
- (4) Was Homer a unicorn? No! There is no such property as the property of being a unicorn.
- (5) Is Homer a permitted author? No! The committee has not yet decided how to apply the censorship rules to the ancient Greeks.

In each of 1–5 it is plausible both that the utterance of ‘No!’ constitutes a denial of the initial sentence, and that the sentence following ‘No!’ is a coherent continuation which says why the denial is correct. But in each case what makes the denial correct is very different in kind. In 1 it is correct because the referent of ‘Homer’ does not satisfy the predicate. In 2 it is correct because ‘Homer’ does not refer. In 3 it is correct because to assert the sentence would be to make a category mistake. In 4 it is correct because the denied sentence’s predicate does not introduce a genuine property: it is a purported natural kind term which does not correspond to any natural kind.²⁶ In 5 the denial is correct because it has not yet been decided what would be required for the subject to satisfy the predicate.²⁷

It is an old observation, prompted by examples like 1–5, that denial seems to be somehow less specific or less informative than assertion: to assert a sentence is to commit yourself to a determinate claim about how the world is; denials seem to carry no such specificity.²⁸ This observation goes part-way towards explaining why philosophers have thought that rules of inference should deal in assertion alone: denials do not seem to be specific enough to be woven into determinate patterns of inference.

And for someone wanting to run the bilateralist defences against the Negation Argument and the Anti-Realist Argument outlined in Section 2, this old objection to allowing denials a place in inferences takes on an acute and distinctive form. The bilateralist responses to the Negation Argument and the Anti-Realist Argument grant the conception of inferences as conserving evidence which drives the Negation Argument, and claim to save classical negation and realism anyway. But the background picture of inferences as conserving evidence is coherent only if each potential premiss or conclusion is associated with a determinate pattern of evidence, so that it is determinate what evidence a sentence brings to an inference as a premiss, and determinate what evidence an inference’s premisses must contain if a sentence is to be delivered as its conclusion. In the terms used to state the harmony constraint, the picture of inference as conserving evidence rests on the idea that each sentence has a ‘direct’ verification. So to uphold a bilateralist model of reasoning against the background view of inference as conserving evidence you would need to say that not only assertions, but also denials, are associated with distinctive patterns of evidence: a denial has a direct ‘verification’ too. But the messiness of denial makes it very hard to see how the notion of a direct verification for a denial could make sense. Given

²⁶ Compare Kripke’s discussion of ‘unicorn’ at (1980, pp. 23–24).

²⁷ Tappenden uses this kind of case to argue that natural language negation is wide scope (‘external’) negation at (1990, Sect. 4).

²⁸ See Horn (2001: ch. 1) for a history of this kind of claim. For versions of the claim in the contemporary literature see Parsons (1984, Sect. 1) and Tappenden (1990, Sect. 3).

denial's messiness, an account of direct verification for a denial would have to be radically disjunctive. The denial of 'Fv' would have to be regarded as containing evidence that *vl* does not satisfy F, or evidence that *v* is empty, or evidence that *l* is not defined for *vl*, and so on. And given the radical kinds of situation in which denial is correct, it is not even clear what the disjuncts would be like. What would a distinctive, or direct, or canonical, verification of the emptiness of a name look like? What about the standard way to establish the ill-definedness of a predicate relative to the singular terms filling its argument places? There are no obvious answers to these questions. So it seems that there is no readily available account of direct verification for a denied sentence. But without an account of direct verification for denials a bilateralist version of the 'inference preserves evidence' story cannot get off the ground.

The plausibility of the claim that there are kinds of statement whose denial conditions are not parasitic on the conditions for their assertion does nothing to undermine the concern that denials are not specific enough to play a role in inference. It may be that denial is independent from assertion, but that denials are just too messy to be systematically inferentially useful.

I take it that of these three problems it is the third that is the most serious. The bilateralist could respond to the first by saying that there are brute linguistic facts, revealed to us by intuition, about which statements' denial conditions are irreducible to conditions for the incorrectness of their assertions. The intuition that knowing that it will never be right to assert that X was snub-nosed does not put you in a position to deny that X was snub-nosed could be claimed to reveal a brute fact of this kind. And there is no obvious bar to addressing the second problem by saying that if *p* is from a class of statements for which a bilateralist model of reasoning is appropriate and *q* from a class which requires a unilateralist model, the model of reasoning for a complex statement built out of *p* and *q* will be unilateralist: *q* drags 'p & q' down with it.²⁹

No such quick solution suggests itself in the case of the third problem—the problem about the messiness of denial. The rest of the paper is about the impact of this problem on the deployments of bilateralism floated in Section 2.2.

I shall start by looking at the extent to which the messiness problem obstructs the more speculative suggestion from Section 2.2—the suggestion that bilateralism might provide the means to overturn the Anti-Realist Argument. Then I shall ask what impact the problem has on the bilateralist response to the Negation Argument considered in its own right, aside from its embedment in the more speculative anti-realist strategy.

3.2 The messiness of denial and the bilateralist response to the anti-realist argument

To what extent does the bilateralist response to the Anti-Realist Argument withstand the problem of the messiness of denial?

As a first step towards answering this question, let us consider the move that I take it a bilateralist must make in the face of the apparent ill-fittedness of natural

²⁹ Compare Dummett on how non-classical operators may be 'superimposed' on classical ones at (1991, pp. 332–337).

language denial to fill the role that bilateralism demands of it. This is to allow that natural language denial is messy, but separate off one kind of ground for denial and say that only denials made on this kind of ground are to be incorporated into a system of inference. Let us say that to deny A on the favoured kind of ground is to make a ‘strong’ denial of A . Then the suggestion would be that ‘ $\neg A$ ’ is to be read as ‘ A is strongly denied’, and that rules of inference treat in assertion and strong denial only.³⁰ If we want to keep the inference-preserves-evidence framework, the most obvious choice of a notion of strong denial for an atomic sentence of form ‘ Φv ’ is denial on the ground that the bearer of v does not satisfy Φ : denial of the kind illustrated by example 1. And the most obvious notion of strong denial for a complex sentence is denial where every singular term in the sentence refers and every predicate is defined for the singular terms which fill its argument places. For if ‘ $\neg A$ ’ is read as ‘ A is strongly denied’, with this notion of strong denial, the claim that ‘ $\neg A$ ’ has a direct or canonical verification is no more problematic than the claim that ‘ $+A$ ’ does. So, according to a proponent of this solution, the objection to bilateralism from the messiness of denial falls away.

But the move to a strong denial version of bilateralism does not save the bilateralist defence of realism that I have floated from the problem of the messiness of denial. And the reason why it does not generalises into a dilemma against which any attempt to deploy bilateralism against the anti-realist argument having granted the messiness of denial will run aground.

Here is why the move to a strong denial system cannot save the bilateralist defence of realism. Recall the structural principle used to derive CNI from the bilateral rules for ‘ \sim ’:

$$+RAA \quad \text{If } X, +A \vdash +B \quad \text{and} \quad X, +A \vdash -B, X \vdash -A.$$

In a strong denial system, the absurdity of A ’s assertion may still entail A ’s denial. But it will not entail A ’s strong denial (it will not entail ‘ $\neg A$ ’). So if ‘ \neg ’ is read as a sign for strong denial, +RAA fails.³¹ But a system in which +RAA fails is a system in which CNI fails too. For let Σ be a bilateralist system within which CNI is valid. So in Σ we have

$$\begin{array}{l} [+A] \\ \vdots \\ \frac{\perp}{+\sim A} \end{array}$$

Now, just as there is room in discussing modal logic for debate about how ‘ \Box ’ and ‘ \Diamond ’ are to be read, and room in discussing tense logic for debate about the readings of ‘ F ’, ‘ P ’, and ‘ N ’, there is room in discussing bilateralist systems for

³⁰ For an example of a ‘strong denial’ bilateralist system see Rumfitt (1997).

³¹ In Rumfitt’s strong denial system +RAA is replaced by a weakened *reductio* rule, whose ‘+’ form is ‘If $X, +A \vdash +B$ and $X, +A \vdash -B$, and $[X]$ determines $[A]$, then $X \vdash -A$ ’, where $[X]$ ‘determines’ $[A]$ iff every valuation that assigns a truth value to every member of X also assigns a truth value to A . See (1997, p. 230).

debate about the readings of ‘+’ and ‘-’. But there are at least two conditions which must be met if ‘-’ is to be a sign for any kind of denial and ‘+’ a sign for assertion. Firstly, ‘ $\neg A$ ’ must be inconsistent with ‘ $+A$ ’: it can never be correct both to assert and to deny a sentence, so if ‘ $+A$ ’ and ‘ $\neg A$ ’ are consistent, ‘+’ and ‘-’ are not signs for assertion and denial. Secondly, ‘ $\neg A$ ’ must be entailed by the assertion of A’s negation: if you can assert ‘ $\sim A$ ’ without incurring commitment to ‘ $\neg A$ ’, either ‘ \sim ’ is not a sign for negation, or ‘-’ is not a sign for denial. So in Σ we will also have

(a) From $+B, \neg B$ infer \perp .

and

(b) From $+ \sim A$ infer $\neg A$. (This is $+ \sim E$.)

Together, CNI and $+ \sim E$ yield:

(c) If $+A \vdash \perp$, infer $\neg A$.

And (a), (c), and the Cut Rule entail

If $+A \vdash +B$ and $+A \vdash \neg B$, infer $\neg A$

which is $+RAA$. So if CNI is valid in a bilateral system $+RAA$ is valid in the system too: a strong denial system (a system in which $+RAA$ fails) cannot accommodate CNI.

A strong denial bilateralist who rejected $+RAA$ and CNI would still be able to keep CNE: the derivation of CNE from the bilateralist rules for ‘ \sim ’ is independent of $+RAA$.³² So a strong denial bilateralist will be able to keep one part of the bilateralist defence of classical negation (the defence of CNE) but not the other (the defence of CNI).

Now recall the move which expands the Negation Argument into the Anti-Realist Argument. This move can be summarised as follows. Given the Manifestation Principle, to defend realism about truth for our language we must show that there are rationally justified aspects of our linguistic behaviour which manifest commitment to realist truth conditions. Given the Bivalence Characterisation of Realism, the most plausible candidate to be an aspect of our linguistic behaviour which manifests commitment to realist truth conditions is the practice of reasoning according to the rules for classical negation: reasoning which takes you from the absurdity of asserting A to the assertion of ‘ $\sim A$ ’, or the absurdity of asserting ‘ $\sim A$ ’ to the assertion of A. So to establish realism about truth you would have to show that these patterns of reasoning are rationally justified.

But these patterns of reasoning employ both CNE and CNI. So to overturn the Anti-Realist Argument while granting the Manifestation Principle and the Bivalence Characterisation you need to show that both CNE and CNI are rationally justified. A defence of CNE which abandons CNI cannot be used to defend realism if the Manifestation Principle and the Bivalence Characterisation are both granted.

³² See Section 2.2 above, and compare Rumfitt (1997, p. 234).

Here is how this argument generalises into a dilemma for the attempt to use bilateralism to overturn the Anti-Realist Argument while granting the elements which generate it.

Any bilateral system will be either a system in which +RAA is valid or one in which it is not. Consider first the systems in which +RAA is not valid. We have shown that these are systems in which CNI is not valid either. So upholding this kind of system does not provide a way to defend realism while granting the connection between realism and classical logic that the Manifestation Principle and the Bivalence Characterisation seem to entail. Now consider the systems in which +RAA is valid. These are systems in which the absurdity of A's assertion entails A's denial. But this claim about the relationship between assertion and denial holds only if by 'denial' we mean 'plain denial', where to make a plain denial of A is just to commit yourself to A's failure to be true. If $\neg A$ is read as A's plain denial, +RAA holds, and CNI is valid in the system. But a bilateralist who reads \neg as a sign for plain denial is committed to treating plain denials as premisses and conclusions in inferences. And according to the conception of inferences as preserving evidence which gives rise to the Negation Argument and the Anti-Realist Argument, and which we are trying to respect, the premisses and conclusions of inferences have direct or canonical means of verification. So to uphold a plain denial system within an inference-preserves-evidence framework is to commit yourself to the conclusion that plain denials have canonical means of verification. But the messiness of denial seems to entail that there is no direct or canonical pattern of evidence for a plain denial. So the systems in which +RAA holds cannot be used to overturn the Anti-Realist Argument either.

Note that it remains open for a bilateralist to reject one or more of the elements that generate the Anti-Realist Argument and look for a connection between strong denial bilateralism and realism within some other framework. For example, someone who thinks that realist truth conditions are verification-transcendent truth conditions, and that commitment to verification-transcendent truth conditions does not entail commitment to bivalence³³ might argue as follows. 'Suppose that strong denial bilateralism provides the right model of reasoning for L. Then reasoning according to CNE is legitimate in L. But to reason according to CNE in L is to allow that truth in L is verification transcendent. For right reasoning must at least preserve truth. And the move from $\neg\neg A$ to A does not preserve verifiability (a proof that A's unprovability cannot be proven falls short of a proof of A). So if right reasoning for our language is reasoning according to a strong denial system, there is a legitimate inferential pattern (reasoning according to CNE) which manifests our commitment to the claim that truth (which is preserved by valid inference) outruns verifiability. Given the Manifestation Principle, it follows that if right reasoning in L is reasoning according to a strong denial bilateralist system, sentences in L have realist truth conditions.' So someone who rejects both the Bivalence Characterisation of Realism and the inference-preserves-evidence framework can provide a plausible argument for the claim that if a strong denial bilateralist model of reasoning is right for our language, we are (legitimately) committed to realism about truth. But this person has moved a long way away from the initial bilateralist

³³ See Section 1.3 above.

response to the Anti-Realist Argument floated at the end of Section 2.2—the response which promises to grant the elements that generate the Anti-Realist Argument and save realism anyway.

So what is the significance of bilateralism for the fight against anti-realism? The argument of the paper so far suggests two possible answers to this question.

The first is that, contrary to the hopes raised in Section 2.2, bilateralism does not bring with it any genuinely new weapons for the fight against anti-realism. According to this answer, when push comes to shove in the attempt to develop a bilateralist response to the anti-realist argument, we are thrown back on a familiar range of options: reject the Bivalence Characterisation; deny that rules of inference must be justified by the inference-preserves-evidence standard; reject the Manifestation Principle; look for behaviours other than reasoning according to CNI and CNE which manifest commitment to realist truth conditions.

The second possibility is to argue that bilateralism does bring a new weapon to the fight against anti-realism, but concede that this weapon is less powerful than the first (overly hopeful) statement of a bilateralist response to the Anti-Realist Argument suggested. On this second view, the right response to the messiness of denial, with respect to a class of statements for which you want to uphold realism, is not to move to a strong denial system. It is to argue that, though we are messy deniers with respect to the language as a whole, when it comes to constructing proofs in the language we retrench to a smaller language fragment within which the kinds of radical failure to be true illustrated by 1–5 do not arise, so within which denial is not messy. The claim might be that our ordinary talk admits of reduction to this special class of statements. Or it might be that the special class of statements is ‘foundational’ in some other way. However the details are to be made out, the claim would have to be that there are grounds for thinking that our ordinary talk somehow rests on or is beholden to a deeper level of language for which a clean model of truth conditions which secures tidy denial holds.

But to say that denial is tidy in L is to uphold the Principle of Bivalence for L . For denial is tidy in L iff there is only one way for sentences in L to fail to be true. And if there is only one way for sentences in L to fail to be true, the sentences in L divide into those which are true and those which fail to be true in the available way—which is to say, into those which are true and those which are false.

So on this second view bilateralism does end up bringing a new weapon to the fight against realism. But it does not deliver on the hopes raised at the end of Section 2.2. Bilateralism does not generate a new way to argue for realism about a given subject matter (by arguing that denial is independent from assertion for statements about the subject matter; that if denial is independent from assertion in L , reasoning according to CNI and CNE are legitimate in L ; and that if reasoning according to CNI and CNE are legitimate in L , sentences in L have realist truth conditions). Rather, bilateralism generates a response to the Anti-Realist Argument which can be deployed only once the old battle to show that our talk about a given subject matter should be regarded as resting on a level of language where sentences have realist truth conditions has already been won.

3.3 The problem of the messiness of denial and the bilateralist defence against logical revisionism

What impact does the problem of the messiness of denial have on the bilateralist response to the Negation Argument, considered aside from the attempt to use this response to overturn the Anti-Realist Argument?

To answer this question it is useful to revisit the standoff between a proponent of the Negation Argument and a classical logician who claims that CNI and CNE can be justified by appeal to the classical truth table. The classical logician is, in effect, supposing the Principle of Bivalence, and offering a derivation of CNI and CNE from this supposition, together with the supposition that ‘ \sim ’ stands for a specific truth function. The supposition of the Principle of Bivalence is implicit in the classical logician’s claims that the rows of a classical truth table exhaust the possibilities with respect to the combinations of truth-values for atomic sentences (A may be and must be either true or false; the pair (A, B) admits of the stipulated four combinations of truth values; the triple (A, B, C) of the stipulated eight; and so on.), and that the complex sentence appearing at the top of the right hand column must be assigned either the value ‘true’ or the value ‘false’ in every row. The classical logician claims that ‘ \sim ’ stands for the truth function represented by its classical truth table; maintains that rules of inference are legitimate iff they preserve truth; and concludes that CNI and CNE are legitimate because they preserve truth given the assignment of truth values that the classical truth table lays down. The proponent of the Negation Argument replies that legitimate rules of inference must preserve evidence; that CNI and CNE do not meet this standard; and that the classical logician’s derivation of CNI and CNE is, therefore, misguided.

From the point of view of this standoff, there is a very quick response that someone proposing the bilateralist response to the Negation Argument might make to the problem of the messiness of denial. This is to point out that a reason to think that denial is messy in L is also, on the face of things, a reason to think that there are multiple ways in which a sentence in L might fail to be true, so that there is at least a good reason to say that denial is messy in L iff the Principle of Bivalence fails for L. Given this claim, it can be argued that the problem of the messiness of denial arises only for language fragments where there is reason to reject the Principle of Bivalence anyway. If we can assume the Principle of Bivalence for L, the full bilateralist response to the Negation Argument outlined in the first part of Section 2.2 is still available for L. So the bilateralist can still weigh in heavily on the classical logician’s side of the standoff about CNI and CNE. The classical logician had argued that if bivalence is assumed CNI and CNE are derivable as legitimate rules of inference. The proponent of the Negation Argument rejected the derivation on the ground that CNI and CNE do not preserve evidence. The bilateralist has shown that in a bilateral system, and given the initial classical supposition of bivalence, CNI and CNE do preserve evidence after all.³⁴

The bilateralist response to the Negation Argument can also preserve part of the classical view of negation for language fragments where denial is messy and only a

³⁴ Compare Rumfitt (2000, p. 811).

‘strong denial’ bilateralist picture is available. We saw in the previous section that CNI fails in ‘strong denial’ systems. But a strong denial bilateralist can still have CNE. And if CNE is still alive, the conception of the negation operator as a ‘switch’ which takes us to a statement’s opposite, from which we can get back to the initial statement by applying the operator again, is alive too. Saving CNE alone does not amount to a partial victory if the aim is to overturn the Anti-Realist Argument. For the patterns of reasoning which (according to a proponent of that argument) manifest commitment to realist truth conditions involve both CNI and CNE. But if the aim is just to fight the Negation Argument, there is no such pressure towards an all-or-nothing account of success. From this point of view, the result that CNE is legitimate in a strong denial system but CNI is not should be interpreted as showing how much of the classical view of negation we should keep for a language fragment where the classical assumption of bivalence is false because denial is messy.

Nevertheless, I do think that the problem of the messiness of denial forces a coda to the bilateralist response to the Negation Argument—a coda which is not captured just by pointing out that the problem arises only for language fragments where there is reason to question the Principle of Bivalence, and that even for these language fragments the response enjoys partial success because a strong denial system saves CNE.

This coda concerns the proposed test for the appropriateness of bilateral models of reasoning.³⁵ This test says that a bilateral model of reasoning is appropriate for L iff denial is independent from assertion in L. The discussion of this section so far suggests that this test needs amendment. The bilateralist should be saying that a bilateral model of reasoning is appropriate for L iff denial is independent from assertion in L, *and* either denial is tidy in L, or there is a suitable notion of strong denial for sentences in L. The amendment will be non-redundant iff there are cases where the independence condition is met but the well-behavedness (tidiness or suitability) condition is not. And there do seem to be cases of this kind.

For example, consider the reasoning involved in developing scientific theories. (I should stress that it is not possible to consider the question of whether this kind of reasoning is bilateral in any depth here. My aim is just to raise an initial concern about the suggestion that it might be.) In scientific reasoning the grounds for denial do seem to outrun those for withholding assertion. For example, a scientific theory might predict the existence of some fundamental particle which has various confirmable properties, and also predict that it will be in principle impossible for us to tell whether these particles have some other property. This theory would give us grounds to think that we will never be in a position to assert that these particles have this property. But these grounds would fall short of grounds for denying that the particles have the property. So, if the independence of denial from assertion is a test for the appropriateness of a bilateral model, we should be bilateralists about scientific reasoning.

But now consider the following humdrum line of scientific thought. Our theory generates hypothesis H. This hypothesis itself generates the prediction that such-and-such experiment will generate a given result. The experiment is performed

³⁵ See note 19.

(many times, by separate, careful, competent groups of experimenters). The predicted result is not obtained. We conclude that H is not true. Either there is something wrong with the reasoning which led us to think that the theory generates H, or the theory itself needs revision.

This line of reasoning does not distinguish between reasons why H is not true—the experiment tells us just that there is something wrong about H and, therefore, that any theory that generates it needs revision; it is silent about exactly what the problem with H might be. So if this pervasive scientific line of thought is rightly modelled as dealing in denial, it must be modelled as dealing in plain denial; it does not treat any one kind of ground for denial as more central than another.

Now consider these examples:

- (6) Does the ether conduct electricity? No! There is no ether.
- (7) Are neutrinos susceptible to the Superweak Force? No! There is no such property as susceptibility to the Superweak Force.³⁶
- (8) Do pions have strange components? No! Our theory does not currently lay down conditions for applying ‘is strange’ to any particles except kaons.³⁷

(6–8) suggest that denial is messy in scientific enquiry.

And now consider the version of the problem of the messiness of denial that arises for a bilateralist who wants to respect the principle that right rules of inference must preserve evidence. Within the scope of this principle, a statement which can serve as a premiss or conclusion in an inference must be associated with a canonical pattern of evidence (so that it will be determinate what evidence the statement brings to a proof when assumed as a premiss, and what evidence must be contained in the evidence brought in by a proof’s premisses if it is to be legitimate to move to the statement as the proof’s conclusion). But if plain denial is messy in L, it is hard to see how there can be canonical verifications for plain denials in L, so hard to see how a bilateralist version of the inference-preserves-evidence story for L can get off the ground.

The solution to this problem that I have canvassed is to restrict the claim that denials can serve as premisses and conclusions in inferences to a special class of strong denials, for which the notion of canonical evidence is not problematic. But scientific reasoning seems to present a case where denial is messy, but where this solution is not available. For the simple, pervasive, and surely legitimate line of thought that I have sketched (from the failure of a prediction generated by an hypothesis to the need to revise the theory from which the hypothesis is derived) turns just on the claim that the tested hypothesis is not true. It is a line of reasoning in which we may engage while recognising that there are many ways that the hypothesis might fail to be true, and while having no idea as to which way of failing to be true we are coming up against. So if this pervasive scientific line of thought is

³⁶ It turned out that there is no Superweak Force.

³⁷ ‘Strangeness’ was initially introduced as a term for whatever property of kaons leads them to display a characteristic pattern of decay. The definition was later revised to recognise pions as containing both ‘strange’ and ‘anti-strange’ components (which cancel each other out, explaining why pions do not display the same pattern).

rightly modelled as dealing in denial, it must be modelled as dealing in plain denial; it does not treat any one kind of reason for denial as more central than another.

So for the case of scientific reasoning the situation seems to be this. Denials are independent from assertions. But if a bilateralist model of reasoning is appropriate it must be a model which allows plain denials as premisses and conclusions in inferences.³⁸ And denial is messy. So we cannot uphold the appropriateness of a bilateral model of reasoning in scientific inquiry within the scope of the principle that right reasoning preserves evidence. And in that case, even though denials in scientific language are independent from assertions, bilateralism does not generate a new defence of reasoning according to CNI and/or CNE in scientific inquiry against the claim that such reasoning is illegitimate because not evidence preserving.

This is not to say that a bilateralist model of reasoning, and a bilateralist justification for classical inference, are not appropriate for the language of scientific inquiry. Maybe it is the inference-preserves-evidence framework that should go instead. But my aim in this paper has been to assess the extent to which bilateralism can deliver on its initial promise of overturning the Negation Argument and the Anti-Realist Argument while granting the basic elements which generate them. And I shall not embark on the further task of questioning these basic elements here.

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³⁸ The model might distinguish plain denial from strong denial, and distinguish two kinds of negation ('external' and 'internal' negation) to coordinate with plain and strong denial. See Rumfitt (1997, pp. 232–234) for an account of what a system which incorporates this distinction might look like. The proposal involves introducing an 'It is true that' operator, so that '–' is still a sign for strong denial, and 'A's plain denial is expressed by '– It is true that A'. But the problem I am raising arises for any bilateralist proposal which allows messy plain denials to be premisses and conclusions in inferences.

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